

Mathematics Lecture Notes for an American Student (1983)

1. Preface

It is almost nine years ago since I taught an American girl student who came to our school, Motomachi Senior High School, as an A.F.S. exchange student from Chicago in the U.S.A. She was admitted into the second grade, but she could not speak Japanese, except for daily greetings.

I was requested to teach her mathematics by Mr. Nakashima, the head teacher of the school. I taught her mathematics on a one to one basis for fifty minutes once a week, from June through September. Every week, I prepared teaching materials which aimed principally at the following items:

- (1) Mathematics in Japan.*
- (2) Japanese tradition related to numbers.*
- (3) Japanese stock phrases related to numbers.*

It was of great use for me to practice English composition. I would like to show you my lecture notes, which I have stowed away in my breast pocket for nine years. Let me add a few more words in conclusion. Special thanks are owed to Ms. Mercedita Roxas who read this report and corrected the errors in English. She is an American teacher of English at our school, Asakita Senior High School. According to her, she is a friend of the above mentioned girl student. They say that this is a small world after all. What a coincidence!

October 14, 1992

2. Mathematics in Japan "Wasan"

Mathematics in Japan had been developed in its own way until the late Meiji Restoration (1868). Originally it came from Europe through China and was developed into the Japanese style which is called "Wasan." One of the oldest mathematics books, we have in Japan, is "Jingoki" which was written by Mitsuyoshi Yoshida who was one of the 'Wasan' educators. It is said that the contents of the book were for the Japanese people at that time (1627).

In the seventeenth century, "Wasan" had its prime epoch under Takakazu Seki (1642~1708) who discovered a mathematical theory of determinants that was more general than Leibniz's and predated it. He is credited with major discoveries in calculus in his own way and wrote them in the book named "Enri." The Tokugawa Shogunate, "Tokugawa Bakufu" took a closed-door policy "Sakoku" to Western countries, except for the Netherlands. They say that we did not have academic contact with Europe at all. So, Japanese mathematicians "Wasan-ka" had developed mathematics in its own way.

Today, I will show you the Japanese inherent way of studying mathematics.

1) Using abacus "soroban"

"Wasan-ka" had used abacus and calculation bars "Sanboku" to calculate and solve equations respectively. We had used "Soroban" in every working office until about ten years ago but we use electronic calculator instead of abacus now.

2) Way of memorization in mathematics

Almost all Japanese people keep the multiplication table in mind, we memorize it by playing upon words like "ni-ni-n-ga shi" (two times two is four) and so forth. As another example, we memorize some square roots like square root two is "Hito-yo Hito-yoni hito-migoro". (One night after night, it is getting in full bloom. 1.41421356 ...)

3. Board of math questions "Sangaku"

Takakazu Seki (1642~1708) had discovered determinants and Newton's approximation at the same time as Gottfried Wilhelm Leibnitz (1646~1716) and Sir Isaac Newton (1642~1727) did them. He made so many contributions to "Wasan" that he would be called a founder of Japanese mathematics.

At that time, "Wasan-ka" mathematicians formed several sects and each sect had mathematical theories as a secret art. Some talented scholars in the sect were awarded teaching certificates which was handed down from a master to a talented disciple. In each sect, they eagerly followed their studies but never taught their results out of the sect. They were proud of their results, thanked a "kami" (god of "Shinto"), probably thought that the results came from natural force, and donated them to the "Shinto" shrines as a board of math questions "Sangaku."

Shintoism is a Japanese traditional religion of which the essence is nature worship. They appeased the kami's anger by giving back what they got from nature. Some shrines have "Sangaku" on their wall. It seems to be laughing at my ignorance just like Sphinx (Greek mythology) asking travellers some riddles in the silent precincts.

4. Zodiac "Junishi"

We have twelve zodiac signs "Junishi" and ten calendar signs "Jikkan" in every day life. The name "Koshi" for example, used in that famous "Koshien" base ball park for high school baseball tournament games, came from the year "Koshi" (甲子), when the lot of the park was reclaimed. These signs came to Japan from ancient China. People thought that political changes would occur periodically and classified them at the time.

"Jikkan" and "Junishi" had ten days and twelve years respectively. Later years, both of them were combined in this order and each combination represents one year.

(1) Signs of "Jikkan"

甲 (Kinoe) 乙 (Kinoto) 丙 (Hinoe) 丁 (Hinoto)
 戊 (Tsuchinoe) 己 (Tsuchinoto) 庚 (Kanoe)
 辛 (Kanoto) 壬 (Mizunoe) 癸 (Mizunoto)

(2) Signs of "Junishi"

子 (Ne rat) 丑 (Ushi bull) 寅 (Tora tiger)
 卯 (U Rabbit) 辰 (Tatsu dragon) 巳 (Mi Snake)
 午 (Uma horse) 未 (Hitsuji sheep) 申 (Saru monkey)
 酉 (Tori domestic fowl) 戌 (Inu dog) 亥 (I wild boar)

Before, I told you that ancient Chinese people believed the cycle of political changes was sixty years with ten "Jikkan" and twelve "Junishi."

What is the meaning of sixty-year cycle in mathematics? It is the least common multiple (L.C.M.) of 10 and 12. There are at least two integers x and y , satisfying the next indeterminate equation:

$$10x = 12y \quad (x \text{ and } y \text{ are mutually prime})$$

$$x = 6, \quad y = 5 \quad \text{then} \quad 10x = 12y = 60$$

Another solution: We are able to take an ordered pair (x, y) , x and y are an element of the ten calendar signs and the twelve horary signs respectively. In addition, both

are odd or even signs at the same time. If they are odd, the number of the ordered pair (x, y) is $5 \times 6 = 30$. If they are even, the number is 30 in the same way. The required number is $30 + 30 = 60$.

5. The rolling picture depicting the Tale of Genji "Genji Monogatari
Emaki" and a perspective

The Japanese classics may not be quite familiar to foreign countries. I will briefly explain to you some of them. The first anthology of "waka" poetry is the "Manyoshu" collection of about 4,500 verses by Japanese from all classes of life from farmers to nobles. It was composed over a nearly 400-year span in the half of the 8th century.

In the 10th century, the first official anthology of waka poetry, the "Kokin Wakashu" was compiled. The words of "Kimigayo" which has been sung by a lot of Japanese as a national anthem, was adopted from it:

*"Kimiga-yowa Chiyoni-yachiyoni Sazare-ishino Iwawoto-narite Kokeno-musumade."
"May thy glorious, glorious reign, Last for ages, myriad ages. Till the tiny pebbles
small, Hoary moss shall overgrow them all." (Excerpt from Saito's Japanese-English
Dictionary)*

*"Thousands of years of happy reign be thine/ Rule on, my lord, till what are pebbles
now/ By ges united to mighty rocks shall grow/ Whose venerable sides the moss doth
line." (Excerpt from The Japan Times 103rd year No.36,060 issue)*

The kana phonetic syllabary had developed from Chinese characters in the 10th century. This made it easy for us to write Japanese sensibilities (a sound of Japanese).

The "Genji Monogatari" was written by lady Murasaki in Japanese kana. It is recognized as the world's first novel, which describes romances of the court nobles and ladies centering around a handsome hero, Genji.

The dawn of the Japanese novel was begun by a lady. Today, all the Senior High School students are studying it in their classics class.

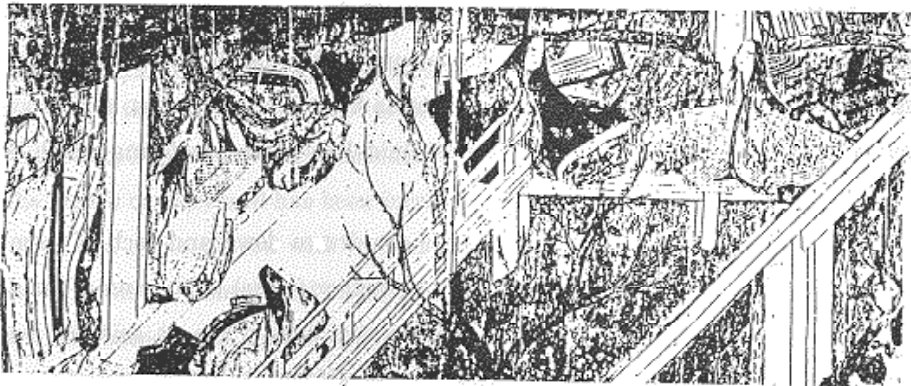
The collecting of romance pictures "Monogatari-e" illustrating such romantic tales

were indispensable objects of indoor hobbies for the nobilities at that time. Lords and noble women competed to collect them.

Most of them have been lost, but a work of the early 12th century still remains. It is the three scrolls in the Reimei-kai (Tokugawa) Collection, and one in the Masuda Collection, of the *Genji Monogatari Emaki* illustrating the *Genji* tales.

They are graceful works brightly painted in pigments of red, vermilion, blue, green and other colors. Their original colors largely remain intact. The original form was horizontal scrolls but they have since been cut into separated sheets. Each one depicts a section of the text and is mounted on wooden frames for the sake of preservation because repeated rolling and unrolling is likely to damage the pictures. The most conspicuous characteristics of these arts are the "Fukinuke yatai" (roofless house) in composition and the "Hiki-me Kagi-hana" (straight line eyes and hooked-line nose) in the portrayal of faces.

(1) *Fukinuke-yatai*:



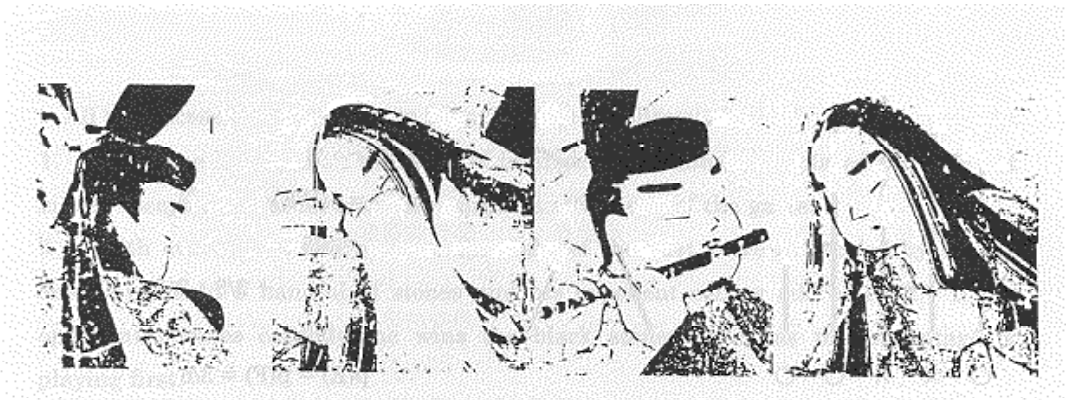
This is a manner of depicting the interior of a house as if viewed from a hole in the ceiling, this unique device is very effective in describing the background of romance scenes. Unlike the western technique of perspective, this method is not a

scientific method of space construction.

(2) *Hikime Kagihana:*

This is an extremely simplified way of portrayal, showing the eyes by a pair of fine lines and the nose by a hook. Eyebrows are added above the eyes, red dots for lips are below the nose and contours of full cheeks make a typical face of the nobilities.

Hikime Kagihana



6. A lottery for supporting money "Amida-kuji" and its probabilities

There are a lot of children's games on numbers. Let me take up one of them, a lottery. In these days, the laughter variety show "Takechan-man" is on TV as one of the most popular programs. A curious song is sung by him on the TV:

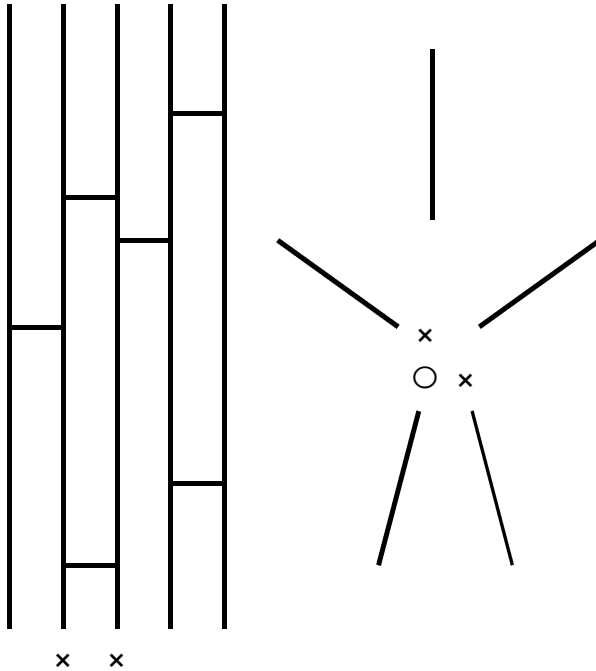
*"Amida-kuji, Amida-kuji Hiite-tanoshi, Amida-kuji Doreni-shiyoka, Amida-kuji"
"Amidakuji", "Amidakuji" It's fun to draw "Amidakuji" I'm wondering which is a
lucky number? "Amidakuji"*

Amidakuji is peculiar to Japan: Some lines are drawn down and each of them is marked a winning number (circle) or a missing number (cross) at the end of lines. As we draw the lots, we add some bars between two lines. You go aside at every branch and trace the line down. Arriving at a circle "Maru," then you win; otherwise you arrive at a cross "Batsu" and you lose.

"Amida-kuji" had been used in the following situation; each of the children brings the same amount of money and they get some candy with the pocket amount. The

children who won a prize in a lottery are given the candy.

The original form of "Amida-kuji" was in the shape of the sunshine. The name "Amida" came from the Buddha's statue which has a halo around its head. What do you think of the scene that some people scramble to draw lots? Let us calculate probabilities of events E and F , and compare them.



Two events E, F are;

E : The first person draws a prize.

F : The second one draws a prize.

$$\begin{aligned}
 p(E) &= \frac{3}{5} \\
 p(F) &= p\left(\frac{F}{E}\right) + p\left(\frac{F}{\bar{E}}\right) \\
 &= \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} \\
 &= \frac{3}{5} \\
 p(E) = p(F) &= \frac{3}{5}
 \end{aligned}$$

Similarly, we can calculate the probabilities of other person's events. Those probabilities are exactly the same as $3/5$.

7. The game of "Go": the national game of Japan

It is said that the game of "go" is the oldest intellectual game in the world. First developed in China over three thousand years ago, the game was introduced to Japan in the eighth century and has been increasing its popularity.

"Go" is a game played with two contestants, placing a black stone and a white stone by turns on a checkered board (Fig.1). The players are usually classed as strong and weak, based upon degree of knowledge and skill. The stronger player takes the white stones and the weaker the black stones. The weaker places some

stones on the dotted points "Hoshi" before the game starts. Black plays first, placing one of his stones on any intersections on the board, thereafter, Black and White play alternately.

The object of "Go" is to secure the territory "Ji" as represented by the number of unoccupied points of intersection (Fig.2). The plans of action are as follows: one is to rail off space by forming walls of stones around the area which the player desires to occupy, and the other is to capture the stones or space of the opponent.

At the end of the game, the amount of vacant space is calculated, and the stones that have been captured are used to fill up the vacant spaces claimed by the opponent.

The rules of the play

All the necessary basic rules to play "Go" are as follows:

- 1. The stone is placed on the point of intersection of lines, once placed they are not moved except when or for counting purposes at the end of the game.*
- 2. With evenly matched players, the right of the first move is decided by having one player hold a handful of stones and his opponent calling odd or even. If the opponent guesses correctly, he wins the black stones, and has the advantage of playing first.*
- 3. When all of the intersections adjacent to a stone or a group of stones are occupied and removed from the board. (Fig.3)*
- 4. It is forbidden to play on any point which is completely surrounded by the opponent's stones and there is only one vacant point, unless the play resulted in the capture of all or part of the surrounding stones, because no stone may remain on the board, if it is completely surrounded by stones of the opposite color.*
- 5. If a group of stones contains two separate "eyes," it is permanently safe from capture, regardless of the enemy stones surrounding the group. (Fig.4)*

Beginners frequently form groups containing false eyes which are called "Kageme." "Kage" means chipped or incomplete. By a false eye is meant a vacant point which must be occupied in order to prevent capture by opponent's completion of a cordon at that point. (Fig.5)

Consider next situation (Fig.6). Assuming that it is white's turn to play, he can

play as shown in (Fig.A) and the opponent result in b (Fig.B). If the players were permitted to take stones by repeating moves in this way, the game would be endless. Those situations are called "Ko"-situation.

6. In a "Ko"-situation, a player cannot repeat his previous move without first playing at some other part of the board.

(Fig.2)

(Fig.3)

(Fig.4)

b

(Fig.5)

(Fig.6)

(Fig.A)

(Fig.B)

Excerpts from SHIGEMI KISHIKAWA: STEPPING STONES TO GO, 1981
(CHARLES E. TUTTLE CO.,)

8. The expression related to number and counting gestures

There are a lot of first names related to numbers, for example, "Taro" (the eldest son), "Jiro" (the second son) and "Saburo" (the third son), that are popular in Japan. Almost all the Japanese have their names which are written in Chinese characters and represent place names. The Chinese characters are ideograms, so that people can easily associate the meaning with the characters. For example, "川" ("Kawa"), we associate a river with its shape, and associate a thick wood with "森" ("Mori")

which has three trees "木" ("Ki"). The expression, to be at sixes and sevens, shows up that six is turning upward and seven is downward: 6767.... This sequence of numbers indicates something to be in disorder.

Japanese people dislike numbers four and nine, because they have the same sounds as death and hardship in Japanese respectively. There is no sickrooms which have numbers four or nine in hospitals in Japan. Hotels or parking lots also have the same tendency as some foreigners dislike number thirteen.

When you count numbers, you may start making a fist and then open it up. In Japan, we open our hand first and then fold the fingers as we go on counting. We indicate two with sticking up forefinger and middle finger at the same time just as turning the peace sign inside out.

I know it is a shameful gesture, once I have read that Winston Churchill showed the sign to the public for the first time when the world war was ended, while a horseman Harvey Smith who won the first prize in the steeple chase, lost his reward £ 2,000 by showing the hateful gesture. Needless to say, Harvey Smith was not aware of his gesture at that time.

We also have a first complex and tend to give some special meanings to the first occasion, for example, a new year's celebration "Oshogatsu," the first product of the season, "Hatsumono" and so on. People's behavior originates from the culture in which they belong to, so it is necessary for us to understand other cultures.

Japanese	English
"Shiku-hakku" To sweat blood
"Ichka-bachika" All or nothing
"Ichikara-jumade" To spell it out for someone
"Junin-toiro" Everyone has his/her taste.
"Nisoku-sanmon" For next to nothing, for chicken feed, for peanuts or for a song
"Nino-ashi-wo-fumu" To have second thoughts (to hesitate)
"Isseki-nicho" To kill two birds with one stone
"Sandome-no-shojiki" Seventh time lucky
"Nidoarukotowa-sando" Things always happen in threes.

"Hyappatsu-hyakuchu"	To hit the mark ten times out of ten
"Gojippo-hyappo"	Six of one and half-dozen of the other
"Hitono-uwasamo-shichiju-gonichi"	A wonder lasts but nine days.
"Roku-daigaku"	Seven Sisters (Barnard, Bryan Mawr, Mount Holyoke, Rad cliff, Smith, Vassal, Welles ley)
"Dai-rokkan"	Sixth sense
"Hyaku-toban"	Dial 999
"Ichi-ichi-kyu"	Dial eleven nine

9. A potted dwarf tree "Bonsai" and abbreviation

According to a vernacular newspaper survey, one fourth of all families are interested in gardening. As you know, so many people, more than a hundred million, are living in this restricted territory. Originally we had a farming people and had lived in a rather spacious country before the rapid economic growth era (1955 ~ 1972) so we are still interested in raising plants, even in such narrow living spaces.

In the process of urbanization under the rapid economic policy by our government, a lot of people are concentrated into the small apartment houses which are often described as 'rabbit hutches' in some foreign papers.

Every spring and fall, twice a year, plant fairs open in every city. People buy plants cheaply and bring them home to take pleasure in raising them. Japanese people appreciate when plants, "Bonsai" or the flower arrangement called "Ikebana," are into their rooms. Though the origin of Bonsai cultivation is not clear, potted trees may have originated in India or China. The object of bonsai cultivation is to produce a small tree that seems to be perfectly natural and to give it an appearance of age. The tree should suggest a landscape or bring to our mind a vivid piece of scenery.

Once I have read that Japanese culture is miniaturization-oriented in the book titled "Chijimi shiko no Nihonjin" (The Compact Culture) by O-Young Lee. He explains that innumerable abbreviations are found in the Japanese language, citing such examples as "Tereko" for tape recorder, "Mazakon" for mother complex,

"Sayonara" for the first half of the expression "If that is so, I must bid you farewell" and "Domo" for the half of "Domo arigato gozaimasu" etc.

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